High-Field Effects in Degenerate Semiconductors: Collision Integral and Isotropic Distribution Function

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In this paper, the authors have derived expressions for the collision integral applicable to degenerate semiconductors and corresponding to various carrier scattering mechanisms. These expressions have been used to obtain expressions for the isotropic part of the carrier-velocity distribution function in different cases of interest. The surfaces of constant energy have been assumed to be spherical, and the energy bands are assumed to be parabolic; this simple model is applicable to a number of semiconductors of interest. The present formulation avoids the use of an effective carrier temperature in the calculation of transport coefficients and other properties of a degenerate semiconductor in the presence of a dc field.

I. INTRODUCTION

A KNOWLEDGE of the collision term in Boltzmann's transfer equation is essential for an adequate analytical investigation of transport phenomena in semiconductors—particularly at high electric fields. In the absence of knowledge of this term for degenerate semiconductors, the hot-carrier phenomenon (at high electric fields) have been analyzed by using the concept of effective carrier temperature, i.e., by assuming the carrier-velocity distribution function to be a Fermi-Dirac distribution function, corresponding to a temperature higher than that of the lattice.[1–3] This approach has led to many interesting results but suffers from the weakness of the basic assumption that the form of the isotropic part of the distribution function remains unchanged on the application of the electric field. In case of nondegenerate semiconductors the results obtained on the assumption of effective carrier temperature are at considerable variance with those obtained by using the appropriate isotropic distribution function obtained by solving Boltzmann's transfer equation with the corresponding collision term.[4] Hence in this paper the authors have presented a derivation of the collision integral appropriate to degenerate semiconductors and corresponding to different collision mechanisms. These expressions for the collision integral may be used to investigate the hot-carrier transport phenomenon in degenerate semiconductors by following techniques analogous to those used for nondegenerate semiconductors.

II. CARRIER-VELOCITY DISTRIBUTION FUNCTION

For nondegenerate semiconductors, the collision term is given by[5]

$$\left(\frac{df}{dt}\right)_c=\int W(\mathbf{v}',\mathbf{v})f(\mathbf{v}')d\mathbf{v}'-\int W(\mathbf{v},\mathbf{v}')f(\mathbf{v})d\mathbf{v}\,,\quad(1)$$

where $W(\mathbf{v}',\mathbf{v})$ represents the probability for the transition of the carrier velocity from v' to v in unit time. For degenerate semiconductors one has to take into account Pauli's exclusion principle which implies that a state already occupied by an electron is not available for occupation to other electrons. Thus for degenerate semiconductors we get

$$\left(\frac{df}{dt}\right)_c=\int W(\mathbf{v}',\mathbf{v})f(\mathbf{v}')[1-f(\mathbf{v})]d\mathbf{v}'-\int W(\mathbf{v},\mathbf{v}')f(\mathbf{v})[1-f(\mathbf{v}')]d\mathbf{v}.\quad(2)$$

To proceed further we need to introduce in Eq. (2) an explicit expression for the transition probability corresponding to the relevant scattering mechanism.

III. ACOUSTICAL-PHONON SCATTERING

In this section we consider the case of scattering of carriers due to the strain caused by the acoustic wave, i.e., the deformation potential scattering. Shockley[6] has shown that if we exclude spin-exchange scattering, the matrix element for the transition is unchanged by the inclusion of Pauli's exclusion principle, when the band-edge shift is linearly dependent on the strain. In the following treatment we shall not consider the spin state of the carriers. Following the procedure adopted by Yamashita *et al.*[7] and after some computation along the conventional lines[8] we obtain

$$\left(\frac{df}{dt}\right)_{\rm ac}=\frac{E_1{}^2m^*}{8\pi^2\rho u_l\hbar^2k}\Bigg(\int_{\phi=0}^{2\pi}d\phi\int_{q=0}^{2k+2m^*u_l/\hbar}dq\times q^2[(N_q+1)f_{\mathrm{k+q}}(1-f_{\mathrm{k}})-N_qf_{\mathrm{k}}(1-f_{\mathrm{k+q}})]+\int_{\phi=0}^{2\pi}d\phi\int_{q=0}^{2k-2m^*u_l/\hbar}dq\times q^2[N_qf_{\mathrm{k-q}}(1-f_{\mathrm{k}})-(N_q+1)f_{\mathrm{k}}(1-f_{\mathrm{k-q}})]\Bigg),\quad(3)$$

[1] R. F. Greene, J. Electron. Control **3**, 387 (1957).
[2] A. Zylbersztejn, in *Proceedings of the Conference on the Physics of Semiconductors, Paris* (Academic, New York, 1964), p. 505.
[3] L. E. Gurevich and I. Ya. Korenblit, Zh. Eksperim. i Teor. Fiz. **44**, 166 (1963) [Soviet Phys. JETP **17**, 1444 (1963)].
[4] E. M. Conwell, *High-Field Transport in Semiconductors* (Academic, New York, 1967), pp. 11–14.
[5] C. Herring, Bell System Tech. J. **34**, 237 (1955).
[6] W. Shockley, *Electrons and Holes in Semiconductors* (Van Nostrand, Princeton, N. J., 1950), p. 538.
[7] J. Yamashita and M. Watanabe, Progr. Theoret. Phys. (Kyoto) **12**, 443 (1954).
[8] Reference 4, pp. 215–20.

where E_1 is the shift of band edge per unit dilation, ρ is the density of the crystal, u_l is the velocity of acoustic wave, $\mathbf{k}$ is the wave vector of electron, $\mathbf{q}$ is the wave vector of phonon, N_q is the number of phonons with wave vector, $\mathbf{q}$ and ϕ is the azimuthal angle of scattering.

Substituting the equilibrium distributions

$$N_q=[\exp(\hbar\omega_q/k_0T)-1]^{-1}$$

and

$$f=[\exp((E-E_F)/k_0T)+1]^{-1}$$

in Eq. (3) one obtains $(\partial f/\partial t)_{\rm ac}=0$ as is expected. k_0 is the Boltzmann constant.

For further analysis we make use of the diffusion approximation

$$f_{\mathbf{k}}=f_0(E)+k_{\mathbf{F}}g(E). \quad (4)$$

Substituting for $f_{\mathbf{k}}$ from Eq. (4) in Eq. (3) and making use of the fact that the phonon energy $\hbar\omega_q$ is generally much smaller than the carrier energy, we may expand $f_{0\mathbf{k}\pm\mathbf{q}}$, $g_{\mathbf{k}\pm\mathbf{q}}$, and $e^{\hbar\omega_q/k_0T}(=e^{xq})$ in a Taylor series. After some further algebraic manipulations, we obtain

$$\left(\frac{df}{dt}\right)_{\rm ac}=\frac{E_1^2m^*}{4\pi\rho u_l\hbar^2k}\left(\int_{q=0}^{2k+2m^*u_l/\hbar}dq\,q^2[f_0(1-f_0)+f_0']\right.$$
$$-\int_{q=0}^{2k-2m^*u_l/\hbar}dq\,q^2[f_0(1-f_0)+f_0']$$
$$+\frac{1}{k_0T}\int_{q=0}^{2k}dq\,q^2(\hbar u_lq)[f_0'(1-2f_0)+f_0'']$$
$$\left.-\frac{k_0T}{\hbar u_lk^2}k_{\mathbf{F}}g\int_{q=0}^{2k}q^3dq\right), \quad (5)$$

where $f_0'\equiv df_0/dx$, $f_0''\equiv d^2f_0/dx^2$, $x=E/k_0T$, and phonon distribution is assumed to be in equilibrium.

After carrying out the integration the final expression for $(df/dt)_{\rm ac}$ has been obtained as follows:

$$\left(\frac{df}{dt}\right)_{\rm ac}=\frac{E_1^2m^*}{4\pi\rho u_l\hbar^2k}\left\{\frac{16m^{*2}Ek_0T}{\hbar^3u_l^{-1}}\right.$$
$$\times\left[\frac{E}{k_0T}f_0''+\left(2+\frac{E}{k_0T}(1-2f_0)\right)f_0'\right.$$
$$\left.\left.+\frac{2}{k_0T}f_0(1-f_0)\right]-\frac{4k_0Tk^2}{\hbar u_l}k_Fg\right\}$$
$$=C_a\frac{1}{\sqrt{x}}\frac{d}{dx}\{x^2[f_0'+f_0(1-f_0)]\}-\nu_{0a}x^{1/2}k_Fg, \quad (6)$$

where

$$C_a=\frac{2\sqrt{2}E_1^2(m^*)^{5/2}(k_0T)^{1/2}}{\pi\rho\hbar^4}$$

and

$$\nu_{0a}=\frac{\sqrt{2}E_1^2m^{*3/2}(k_0T)^{3/2}}{\pi\rho u_l^2\hbar^4}.$$

On the other hand, $(df/dt)_{\rm field}$ is written as usual,[8]

$$\left(\frac{df}{dt}\right)_{\rm field}=\frac{eF}{\hbar}\left[\frac{2}{3}\frac{1}{\sqrt{x}}\frac{d}{dx}(x^{3/2}g)+\frac{\hbar^2k_F}{m^*k_0T}f_0'\right], \quad (7)$$

where $-e$ is the electronic charge and F is the externally applied dc field. The Boltzmann's transfer equation for the equilibrium state is

$$\left(\frac{df}{dt}\right)_c+\left(\frac{df}{dt}\right)_{\rm field}=0. \quad (8)$$

Substituting from the Eqs. (6) and (7) in Eq. (8) and separating isotropic and anisotropic parts, we obtain

$$\frac{C_a}{\sqrt{x}}\{x^2f_0''+[2x+x^2(1-2f_0)]f_0'+2xf_0(1-f_0)\}$$
$$+\frac{2eF}{\hbar}\frac{1}{\sqrt{x}}\frac{d}{dx}(x^{3/2}g)=0 \quad (9)$$

and

$$\nu_{0a}x^{1/2}g=\frac{eF\hbar}{m^*k_0T}f_0'. \quad (10)$$

From Eqs. (9) and (10) we obtain

$$(x+p)f_0''+[2+x(1-2f_0)+p/x]f_0'$$
$$+2f_0(1-f_0)=0, \quad (11)$$

where

$$p=\frac{2e^2F^2}{3\nu_{0a}C_am^*k_0T}.$$

The solution of Eq. (11) is

$$f_0=[e^{[x-\eta-p\ln(x+p)]}+1]^{-1}, \quad (12a)$$

where η is a constant, and it can be interpreted as dimensionless Fermi energy E_F/k_0T. An additive constant on the right-hand side of the expression for f_0 given by Eq. (12a) is taken to be zero, so as to satisfy the condition that f_0 vanishes when x tends to infinity.

For the nondegenerate case $\eta\rightarrow-\infty$, we may neglect 1 in comparison of $e^{[x-\eta-p\ln(x+p)]}$, and hence

$$f^0\propto(x+p)^pe^{-x}, \quad (12b)$$

which is the result obtained by Yamashita *et al.*[7] In the case of weak dc fields p can be seen to be much smaller than the average value of x, and hence we may write

$$f_0=f_0(p=0)+p\left(\frac{\partial f_0}{\partial p}\right)_{p=0}$$
$$=\frac{1}{e^{x-\eta}+1}-\frac{p\ln xe^{x-\eta}}{(e^{x-\eta}+1)^2}. \quad (13)$$

In the case of high fields p is much greater than the average value of x and hence Eq. (11) can be approximated by

$$pf_0''+[x(1-2f_0)+p/x]f_0'+2f_0(1-f_0)=0$$

or

$$f_0=(e^{(x^2-\eta^2)/2p}+1)^{-1} \tag{14}$$

IV. PIEZOELECTRIC SCATTERING

Laikhtman[9] has obtained an expression for the distribution function of carrier velocities in a nondegenerate piezoelectric semiconductor subjected to a dc field. In this section his analysis has been modified to take into account Pauli's exclusion principle so that the result will be valid for degenerate piezoelectric semiconductor. The matrix element for either emission or absorption of phonons is taken to be the same as that for the nondegenerate case, as was done for the acoustic mode in Sec. III. Thus we find

$$\left(\frac{df}{dt}\right)_{\text{pe}}=\frac{C_{pe}}{\sqrt{x}}\frac{d}{dx}\{x[f_0'+f_0(1-f_0)]\}-\nu_{0pe}x^{-1/2}k_Fg\,, \tag{15}$$

where

$$C_{pe}=\frac{16\sqrt{2}\pi(m^*)^{3/2}u_l^2e^2\beta^2}{\rho\chi^2\hbar^2(k_0T)^{1/2}}\,,$$

$$\nu_{0pe}=\frac{8\sqrt{2}\pi m^{*1/2}e^2\beta^2(k_0T)^{1/2}}{\rho\chi^2\hbar^2}\,,$$

χ is the dielectric constant, ρ is the density of the semiconductor, β is the piezoelectric tensor, and u_l is the velocity of sound. β is assumed isotropic for simplicity in the analysis. From Eqs. (15), (7), and (8), we obtain

$$\frac{C_{\text{pe}}}{\sqrt{x}}\frac{d}{dx}\{x[f_0'+f_0(1-f_0)]\}+\frac{2}{3}\frac{eF}{\hbar}\frac{1}{\sqrt{x}}\frac{d}{dx}(x^{3/2}g) \tag{16}$$

and

$$\nu_{0pe}x^{-1/2}g=\frac{eF\hbar}{m^*k_0T}f_0'\,. \tag{17}$$

Eliminating g between Eqs. (16) and (17), we obtain

$$(1+p_ex)f_0'+f_0(1-f_0)=0$$

or

$$f_0=\{\exp[p_e^{-1}\ln(1+p_ex)-\eta]+1\}^{-1}, \tag{18a}$$

where

$$p_e=2e^2F^2/3\nu_{0pe}C_{pe}m^*k_0T\,.$$

For nondegenerate semiconductors $\eta\rightarrow-\infty$ and hence Eq. (18a) reduces to

$$f^0\propto(1+p_ex)^{-1/p_e} \tag{18b}$$

which is the result obtained by Laikhtman.[9] It has been pointed out by Laikhtman[9] that f^0 can be normalized only for fields F smaller than some fixed valued determined by the parameters of the semiconductor. The case with f_0 defined by Eq. (18a) is similar.

For the case of n-InSb and in the temperature range 2–10°K piezoelectric scattering mixed with acoustical-phonon scattering explains very well the transport properties.[2,10] The complete collision integral in this case is given by

$$\left(\frac{df}{dt}\right)_c=\left(\frac{df}{dt}\right)_{\text{ac}}+\left(\frac{df}{dt}\right)_{\text{pe}}. \tag{19}$$

From Eqs. (8), (19), (6), (15), and (7) we obtain

$$\frac{C_a}{\sqrt{x}}\frac{d}{dx}\{x^2[f_0'+f_0(1-f_0)]\}+\frac{C_{\text{pe}}}{\sqrt{x}}\{x[f_0'+f_0(1-f_0)]\}+\frac{2}{3}\frac{eF}{\hbar}\frac{1}{\sqrt{x}}\frac{d}{dx}(x^{3/2}g)=0 \tag{20}$$

and

$$\nu_{0a}x^{1/2}g+\nu_{0pe}x^{-1/2}g=(eF\hbar/m^*k_0T)f_0'\,. \tag{21}$$

Eliminating g between Eqs. (20) and (21) we obtain

$$\left(x+C_1+\frac{p}{1+bx^{-1}}\right)f_0'+(x+C_1)f_0(1-f_0)=0$$

or

$$f_0=(e^{x-\eta-I_1}+1)^{-1}, \tag{22}$$

where

$$C_1=\frac{C_{\text{pe}}}{C_a},\quad b=\frac{\nu_{0pe}}{\nu_{0a}},$$

$$I_1=\int^x\frac{p\,dx}{(1+bx^{-1})(x+C_1)+p}$$

$$=\frac{p}{(4bC_1-A^2)^{1/2}}\times\tan^{-1}\frac{(2x+A)}{(4bC_1-A^2)^{1/2}},\quad\text{for }(A^2-4bC_1)<0$$

$$=-\frac{pA}{2x+A},\quad\text{for }(A^2-4bC_1)=0$$

$$=\frac{p}{2\sqrt{A^2-4bC_1}}\times\ln\left\{\frac{2x+A-\sqrt{A^2-4bC_1}}{2x+A+\sqrt{A^2-4bC_1}}\right\},\quad\text{for }(A^2-4bC_1)>0$$

and

$$A=(b+p+C_1)\,.$$

[9] B. D. Laikhtman, Fiz. Tverd. Tela 6, 3217 (1964) [Soviet Phys. Solid State 6, 2573 (1965)].

[10] R. J. Sladek, Phys. Rev. 120, 1589 (1960).

V. IONIZED IMPURITY SCATTERING

Yamashita[11] and Sanchez[12] have studied the warm-electron properties of heavily doped n-Ge using the concept of an effective carrier temperature. To investigate the effect of ionized impurity scattering on the isotropic part of carrier-velocity distribution function, we make use of the fact that this scattering is essentially elastic, and hence we may neglect its contribution to $(df/dt)_c$. For spherical constant-energy surfaces there exists a relaxation time τ_i given by Conwell-Weisskopf formula[13] as follows:

$$\tau_i=\frac{\sqrt{2}}{\pi}\frac{m^{*1/2}\epsilon_s{}^2E^{3/2}}{e^4N_iG(E)},$$

where $G(E)=\ln[1+(\epsilon_s/e^2N_i{}^{1/3})^2E^2]$, ϵ_s is the dielectric constant of the medium, and N_i is the concentration of ionized impurity. Whenever degeneracy is achieved by lowering the temperature of the crystal and by light doping, $G(E)$ is universally approximated by a constant and hence $\tau_i\propto E^{3/2}$. But as discussed by Blatt[14] and Conwell[15] for high and low temperatures, the function $G(E)$, though slowly varying, should not be approximated by a constant. In this case it will be more appropriate to take $\tau_i\propto E$ which we shall consider in the following treatment, and thus write

$$\nu_i=\nu_{0i}x^{-1}, \tag{23}$$

where

$$\nu_{0i}=\frac{\pi e^4N_i}{\sqrt{2}(m^*)^{1/2}\epsilon_s{}^2(k_0T)^{3/2}}.$$

In the case of energy relaxation of the carriers by deformation potential scattering the effective collision frequency can be written as

$$\nu=\nu_{0a}x^{1/2}+\nu_{0i}x^{-1}=\nu_{0a}x^{1/2}(1+dx^{-3/2}), \tag{24}$$

where

$$d=\nu_{0i}/\nu_{0a}.$$

From Eqs. (8), (6), (24), and (7) we obtain Eq. (9) and

$$\nu_{0a}x^{1/2}(1+dx^{-3/2})g=\frac{eF\hbar}{m^*k_0T}f_0'. \tag{25}$$

Eliminating g between Eqs. (9) and (25), we obtain

$$\left(x+\frac{p}{1+dx^{-3/2}}\right)f_0'+xf_0(1-f_0)=0 \tag{26}$$

or

$$f_0=(e^{x-\eta-I_2}+1)^{-1},$$

where

$$I_2=p\int^x\frac{x^{1/2}dx}{x^{3/2}+px^{1/2}+d}.$$

In the case when

$$\nu_i=\nu_{0i}x^{-3/2},\quad I_2=p\int^x\frac{xdx}{x^2+xp+d}.$$

For the case of nondegenerate semiconductor, f_0 given by Eq. (26) reduces to

$$f^0\propto e^{-x+I_2}.$$

In the absence of ionized impurity scattering, i.e., $d=0$, f_0 reduces to the form given by Eq. (12b), first derived by Yamashita *et al.*[7]

VI. NONPOLAR OPTICAL-PHONON SCATTERING

For the case of n-Ge the energy loss of carriers to the optical mode of lattice is important at room temperature and even for low fields. The following considerations will apply at temperatures greater than the Debye temperature and at any value of heating field or at the lower temperature, but with sufficiently high heating fields such that the mean energy of the carriers is greater than the optical-phonon energy. We take the matrix element for either emission or absorption of the optical phonon, such as in the case of nondegenerate semiconductors, as was done for the acoustical-phonon scattering in Sec. III.[16] Including Pauli's exclusion principle we obtain the following collision term for nonpolar optical mode of the lattice scattering:

$$\left(\frac{df}{dt}\right)_{\rm op}=\frac{C_{\rm op}}{\sqrt{x}}\frac{d}{dx}\{(e^{x_0}+1)x_0xf_0''+[(e^{x_0}+1)x_0+2(e^{x_0}-1)x(1-2f_0)]f_0'+2(e^{x_0}-1)f_0(1-f_0)\}-\nu_{\rm op}x^{1/2}k_Fg, \tag{27}$$

where $x_0=\hbar\omega_0/k_0T$, the dimensionless energy of the optical phonon, and

$$C_{\rm op}=\frac{(m^*)^{3/2}E_{1\rm op}{}^2\omega_0{}^2}{2\sqrt{2}\pi\hbar^2\rho u_l{}^2(k_0T)^{1/2}}\frac{1}{e^{x_0}-1},$$

$$\nu_{\rm op}=\frac{(m^*)^{3/2}(k_0T)^{3/2}E_{1\rm op}{}^2}{\sqrt{2}\pi\hbar^4\rho u_l{}^2},\quad E_{1\rm op}{}^2=\frac{D_t{}^2K^2u_l{}^2}{\omega_0{}^2},$$

D_t is the coupling constant between electrons and nonpolar optical mode of lattice vibration, and K is the first reciprocal vector of the lattice. Calculations including all the three processes (e.g., ionized impurity scattering, acoustical-phonon scattering and optical-phonon scattering) were carried out by Sanchez[12] for fields from zero to several kV/cm for highly doped germanium

[11] J. Yamashita, The University of Tokyo, Japan, Technical Report Ser. A.N.O. 5, 1960 (unpublished).
[12] M. Sanchez, Solid-State Electron. 6, 183 (1963).
[13] E. M. Conwell and V. F. Weisskopf, Phys. Rev. 77, 388 (1950).
[14] F. J. Blatt, Phys. Rev. 105, 1203 (1957).
[15] Reference 4, p. 185.
[16] Reference 4, p. 150.

using a Maxwell-Boltzmann distribution function with effective carrier temperature. We can explicitly evaluate the form of carrier-velocity distribution function as follows by including Pauli's exclusion principle. From Eqs. (27), (6), (23), (8), and (7) we obtain

$$\frac{C_a}{\sqrt{x}}\frac{d}{dx}\{x^2[f_0'+f_0(1-f_0)]\}+\frac{C_{\text{op}}}{\sqrt{x}}\frac{d}{dx}\left\{x\left[f_0'+\tfrac{1}{2}x_0\frac{e^{x_0}+1}{e^{x_0}-1}f_0(1-f_0)\right]\right\}(e^{x_0}-1)+\frac{2}{3}\frac{eF}{\hbar}\frac{1}{\sqrt{x}}(x^{3/2}g)=0 \quad (28)$$

and

$$\nu_{0a}x^{1/2}(1+a+dx^{-3/2})g=\frac{eF\hbar}{m^*k_0T}f_0', \quad (29)$$

where

$$a=\nu_{0p}/\nu_{0a}.$$

Eliminating g between Eqs. (28) and (29) we obtain

$$\left(x+C_2+\frac{p}{1+a+dx^{-3/2}}\right)f_0'+\left(x+C_2\frac{x_0}{2}\frac{e^{x_0}+1}{e^{x_0}-1}\right)f_0(1-f_0)=0 \quad (30)$$

or

$$f_0=\frac{1}{e^{x-\eta-I_3}+1},$$

where

$$C_2=\frac{C_{\text{op}}(e^{x_0}-1)}{C_a},$$

$$I_3=\int^x\frac{p+C_2(1-\frac{1}{2}x_0(e^{x_0}+1)/(e^{x_0}-1))(1+a+dx^{-3/2})}{p+(x+C_2)(1+a+dx^{-3/2})}dx$$

$$=\int^x\frac{p\,dx}{(1+a+dx^{-3/2})(x+C_2)+p}, \quad \text{for } x_0\ll 1.$$

VII. CONCLUSIONS

The above formulation determines the form of the carrier-velocity distribution function in degenerate semiconductors in the presence of a dc field. It avoids the use of effective carrier temperature in the calculation of transport and other properties of degenerate semiconductors.

The next part of this series will deal with the investigation of transport phenomena at high electric fields, using the collision integral and carrier-velocity distribution function derived in this investigation.

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